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Title:

PHYSICAL INTREPRETATION OF MATHEMATICALLY INVARIANT K(ρ,P) TYPE EQUATIONS OF STATE FOR HYDRODYNAMICALLY DRIVEN FLOW

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PHYSICAL INTREPRETATION OF MATHEMATICALLY INVARIANT K(ρ,P) TYPE EQUATIONS OF STATE FOR HYDRODYNAMICALLY DRIVEN FLOW

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In order to apply the power of a full group analysis[1] to the problem of an expanding shock in planar, cylindrical, and spherical geometries, the expression for the shock front position R[t] has been modified to allow the wave to propagate through a general non-uniform medium. This representation incorporates the group parameter ratios as meaningful physical quantities and reduces to the classical Sedov-Taylor solution for a uniform media.
Expected profiles for the density, particle velocity, and pressure behind a spherically diverging shock wave are then calculated using the Tait equation of state for a moderate (i.e., 20 t TNT equivalent) blast load propagating through NaCl. The changes in flow variables are plotted for Mach ≤ 1.5
Finally, effects due to variations in the material uniformity are shown as changes in the first derivative of the bulk modulus (i.e., K_o').

INTRODUCTION

In the companion paper[1] a general solution to the 1D hydrodynamic shock wave was given in terms of its group invariance properties.
Self-similar profiles for the reduced density, particle velocity, and pressure behind the shock were shown to be explicit functions of the Mach number at the front, the equation of state, the shock formation time, and the uniformity of the material ahead of the shock.
In order to illustrate how the group theoretic method can be applied to the investigation of real experiments, this study will consider a moderately strong (M≤1.5), spherically diverging shock wave propagating through a solid block of NaCl using the Tait equation of state

DIMENSIONAL ANALYSIS APPLIED TO THE EXPANDING SHOCK FRONT

The traditional Taylor-Sedov dimensional expression for the expanding shock front only allows for an ideal gas, power law non-uniformity.
In order to apply the present work into the non-uniform regime for a general material, it is necessary to incorporate a velocity dependence into the expression for the expanding shock front, R[t].
This yields the following ordinary differential equation;

$$R[t] = (\rho^{-1} E R'[t]^{\mu/\lambda - 1})^{(1-\lambda)/(J+1)} (t-\sigma)^{\lambda} \quad (1)$$

Where R'[t] is the velocity of the shock front, t is time, ρ is the density, E is the energy of the blast, μ is the uniformity of the material ahead of the

shock, λ is the expansion rate of the shock, σ the shock formation time, and j=0, 1, or 2 for rectangular, cylindrical, and spherical geometries respectively. Note that this equation reduces to the classical solution for a uniform material.
This differential equation has the solution

$$R[t] = 2^{\Delta\mu/2(1+J)} \left(\frac{E}{\rho}\right)^{1/(3+J)} (t-\sigma)^{-\Delta\lambda/2} \times \\ ((J+3)(t-\sigma-(2\sigma)^{-2(\lambda-1)/\mu}(t-\sigma)^{-\lambda(1+J)/\mu}))^{1+\Delta/2} \quad (2)$$

Where Δ=2(J+1)/[(J+3)(λ-1)] and the shock position has been shifted so that R[t]=0 at t=σ (the initial point where the shock wave forms).
The group parameters, λ and μ, are related through the invariance of the energy integral (i.e., μ=2-(J+3)λ) and both are ultimately functions of the properties of the propagating medium
Figures 1 through 3 are plots of the un-scaled (i.e., the E/ρ term has been taken out) shock front position, velocity, and acceleration respectively. Note that the shock wave takes a finite amount of time to form.

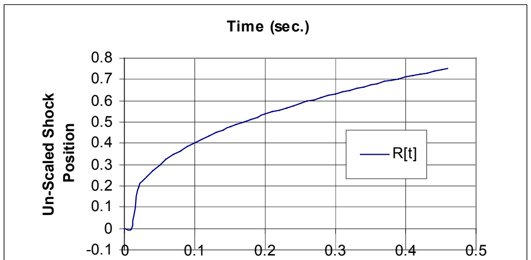

Figure 1 Shock Front Position

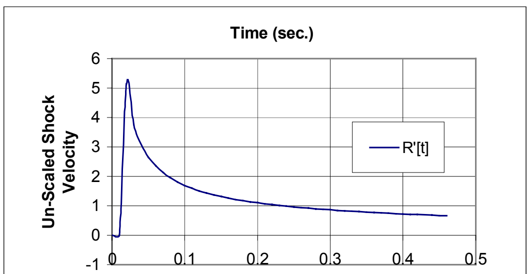

Figure 2 Shock Front Velocity

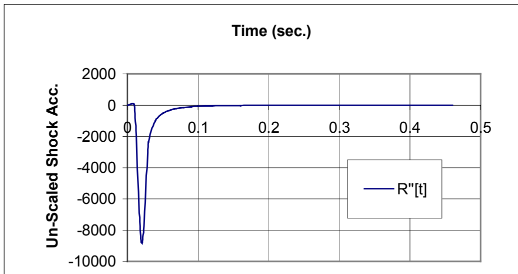

Figure 3 Shock Front Acceleration

ILLUSTRATIVE EXAMPLE

As an example, consider the Tait Equation of State[2] with a shock propagating through NaCl (ρ=2.17 gm/cc, c=2440m/s, Ko=23.81GPa and Ko'=5.68)[3].
We will assume that the shock wave is produced by a 20 ton TNT equivalent blast propagating through a uniform block of NaCl and that the shock takes 5ms to form.
The choice of such a long formation time is to allow graphical illustration of the shock formation delay on the same graph as the characteristic shapes of the shock front position over a reasonably large range of time. Actual formation times would of course be much shorter.
Figure 4 shows the speed of the shock front as a function of time for such a blast.

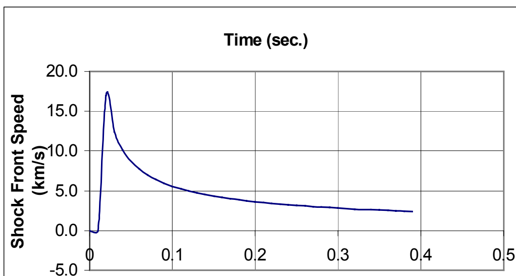

Figure 4 u/g[x] For 20 tons TNT

From this calculation a correspondence between the shock speed and time can be determined for the blast and is shown as Table 1.

Table 1. Shock Speed for a 20t TNT Equiv. Wave Passing Through NaCl

Time after Initiation (s)	Mach No. (M)
0.20	1.5
0.28	1.2
0.33	1.1

Each Mach number corresponds to a unique compression ratio, β, [Ref. 1, Eq. 30] at the shock front. The magnitude of the compression is dependent on the particular equation of state (e.g., Tait). Following the methodology outlined in Reference 1, this compression ratio is used to generate the initial conditions at the front and the reduced equations for f[ξ], g[ξ], and h[ξ] are solved numerically from the coupled set of equations [Ref. 1, Eqs. 11-17].

After calculating these reduced profiles, the actual flow variables are recovered using the following scales;

For the density

$$\rho(\xi,t) = \rho_o(t) f(\xi) \tag{3}$$

and the velocity

$$u(\xi,t) = R'[t] g(\xi) \tag{4}$$

and finally the pressure

$$P(\xi,t) = \rho_o(t) R'[t]^2 h[\xi] \tag{5}$$

This analysis has been carried out in generating figures 5 - 7.

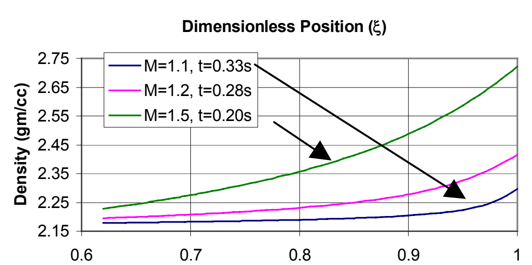

Figure 5 Density Profiles for NaCl

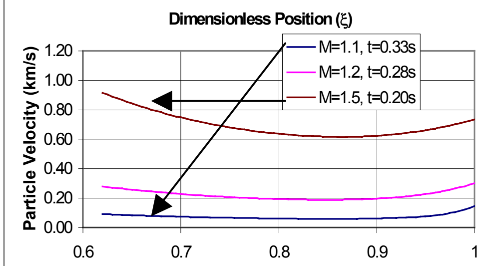

Figure 6 Velocity Profiles for NaCl

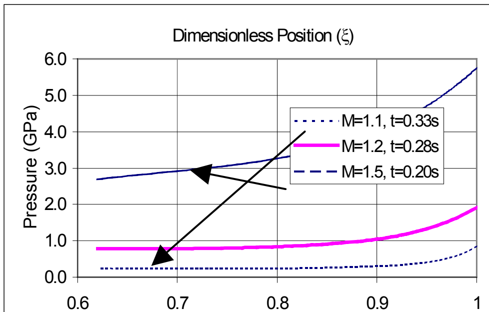

Figure 7 Pressure Profiles for NaCl

CHANGES IN Ko' FOR NaCl

Group theory may also be used to aid in the interpretation of experiments.

As a case in point, Chbabildas and Ruoff[3] reported values for the Ko' of NaCl between 5.68 and 5.98. By performing the procedure outlined in this paper, a researcher may be able to find an inconsistency in their experiment by determining the density, particle velocity, and pressure profiles behind the shock for a specified Mach number using the group theory representation of the flow.

This is because differences in the numerical value of the material parameters and the uniformity of the material in front of the shock show up quite clearly as variations in the calculated density, particle velocity, and pressure profiles. These differences are unique signatures that modify each profile as the shock slows down.

To illustrate this point, calculations for the reduced flow variables have been calculated for the range of values for NaCl stated above.

Figures 8-10 show how the density, particle velocity, and pressure profiles vary due to changes in the material uniformity parameter μ, and the material parameter Ko', for a general spherically diverging shock wave for the asymptotically limiting compression ratio of 1.68 in NaCl.

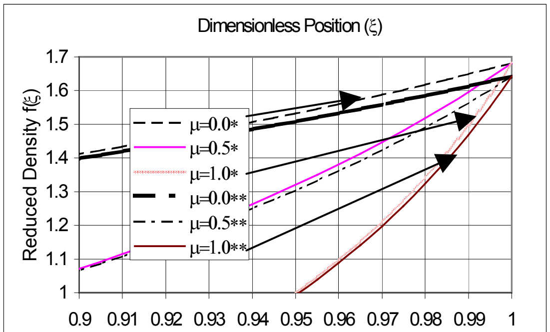

Figure 8 NaCl Variations for Ko' 5.68* and 5.98**

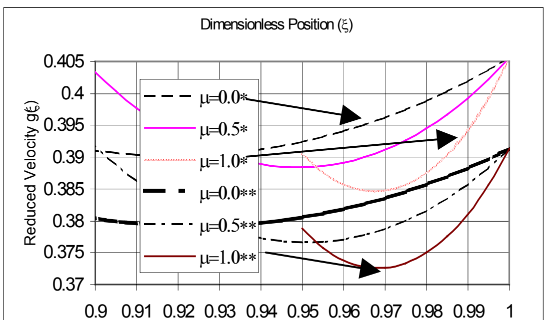

Figure 9 NaCl Variations for Ko' foy 5.68* & 5.98**

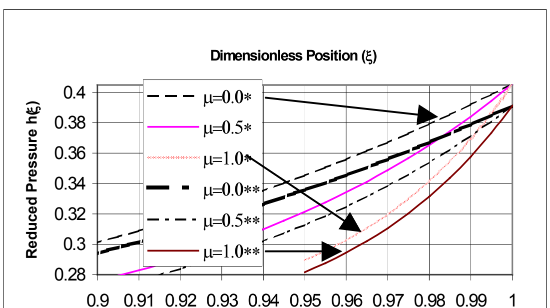

Figure 10 NaCl Variations for Ko' for 5.68* and 5.98**

DISCUSSION AND CONCLUSIONS

The application of this method is straightforward and powerful. It does not require that the total evolution of the flow be determined.
To due this, the initial conditions on the backside of the shock front are computed from the Mach/compression correlation at the front. Experimentally, this correlation would be made through regression of the shock front position with time (i.e., R[t]).
The Mach number of the front is then used to estimate the compression ratio for a given equation of state at a particular moment of time.
The flow variables can then be determined via quadrature of the reduced equations and scaled.

REFERENCES

(1) Hrbek, G. M., in *Proceedings Shock Compression of Condensed Matter -**2001*** (H1.078), (2001).
(2) Kushwah, S.S., and J. Shanker, *Physica B* **253**, 90-95 (1998).
(3) Chbabildas, L. C., and A.L. Ruoff, *J. of Appl. Phys.* **47** (B13), 4182-4187 (1976).